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Revisiting "A theory of debt maturity and innovation" [J. Econ. Theory 218 (2024) 105828] ⋆

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ABSTRACT

We reconsider the incomplete contracting framework of Mitkov (2024) in which an entrepreneur seeks funding for a project and can privately choose its type. Renegotiation is permitted at an intermediate date. We characterize the conditions under which pay-for-failure is necessary to incentivize the entrepreneur to undertake the value-maximizing innovative project. When pay-for-failure is necessary, we generalize Mitkov (2024)'s results by showing that callable long-term debt can replicate the state-contingent contract for a strictly positive measure of parameters, and hence, we provide a rationale for its use in this setting.

1. Introduction

How should entrepreneurs be induced to invest in innovative projects when state-contingent contracts are not permitted? Standard high-powered incentives may fail to effectively induce investment in innovation, highlighting the need for alternative mechanisms such as pay-for-failure (see, e.g., Manso 2011). In this context, Mitkov (2024) examines a model where contracts are incomplete and ex post renegotiation of the original contract is possible at an intermediate date (as in Grossman and Hart 1986). The main insight in Mitkov (2024) is that with (callable) long-term debt entrepreneurs have the right to either continue or liquidate the project at the intermediate date. If the project is in the failure state at this date and financiers would like to enforce liquidation, they must compensate the entrepreneur to induce the efficient decision, thereby creating pay-for-failure.

Mitkov (2024) looks for the most cost-efficient way to implement the novel action and then allocates the ex ante surplus through lump-sum transfers from investors to the entrepreneur. At this allocation, at least one (if not both) of the two incentive constraints faced by the entrepreneur is binding. We depart from Mitkov (2024) to allow for any contract which satisfies investors' zero profit condition (of which the least cost contract with ex ante transfers is a subset). We show that restricting attention to the least cost contract is not without loss of generality in this setting: there exist equilibria in which both the incentive constraints may be simultaneously slack.

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Our approach generalizes the results in Mitkov (2024) and delivers two new implications: First, in Mitkov (2024), pay-for-failure is required to induce investment in the novel project whenever it is highly likely to fail. In this case, he claims that long-term debt must be used to induce investment in the novel project. In contrast, we show that pay-for-failure is required only when the novel project is both highly likely to fail and not very profitable; in all other cases, short-term debt can induce investment in the novel project without pay-for-failure. Second, when pay-for-failure is necessary, callable long-term debt implements the optimal state-contingent contract for a strictly positive measure of the values of the exogenous bargaining power (with respect to the split of the liquidation proceeds) between the two parties as opposed to only one single value (zero measure) in Mitkov (2024). Thus, we provide a rationale for the use of callable long-term debt in this setting.

2. Model set-up

There are three dates $t \in \{0,\ 1,\ 2\}$, universal risk-neutrality, and no time-discounting. At $t = 0$, an entrepreneur has no funds of her own and seeks I units of funds from deep-pocketed (competitive) external investors. She uses these funds to invest in a project and chooses one of three projects – shirking, standard project, or novel project. The entrepreneur's choice of project determines the probabilities with which the project is in the good (G), middle (M), or bad (B) state at $t = 1$. The project generates a payoff at $t = 2$. The payoff depends on the state of the project at $t = 1$. Specifically, it produces Y with probability q_i and 0 with probability $1 - q_i$, where $i \in \{B,\ M,\ G\}$. The project can be liquidated at $t = 1$ for a value $L < I$. We assume that liquidation is efficient only in the bad state, $q_G Y > q_M Y > L > q_B Y$. The probability distribution over states is $p \equiv (p_B, p_M, p_G)$. If an entrepreneur shirks, she obtains a private benefit $b \geq 0$ but it implies that the state is bad with probability, 1, i.e., $p = (1, 0, 0)$. The standard project has no private cost or benefit and implies $p = (0, \alpha_M, \alpha_G)$. The choice of the novel project implies a private cost $c > 0$ and $p = (\beta_B, 0, \beta_G)$. The project's state is observable at $t = 1$, but not contractible. We assume that shirking is value-destroying, while the novel project produces a strictly positive surplus, S_{novel}, which exceeds the surplus generated by the standard project.

The contract at $t = 0$ between the investor and the entrepreneur specifies the allocation of both cash flow rights and control rights. The party holding control rights decides whether to liquidate the project at $t = 1$. Renegotiation between contracting parties is allowed, ensuring that liquidation is always efficient: a project is liquidated only in the bad state. Liquidation yields L, with the bargaining protocol allocating a fraction μ of the surplus to the entrepreneur and a fraction $(1 - \mu)$ to the investor where $\mu \in [0, 1]$.

In the first-best, at $t = 0$, the entrepreneur pays the setup cost, I, and chooses the novel project since it generates the maximum surplus, and at $t = 1$, the project is liquidated only in the bad state and continued otherwise.

We consider the following game: At Stage 1 $(t = 0)$, an entrepreneur seeks I units to invest in a project. Competitive investors propose a contract specifying the promised repayment $R_{0,t}(i)$ at date t in state i, subject to limited liability; if state-contingent contracts are not allowed, the promised repayment is simply $R_{0,t}$. At Stage 2, after securing funds and incurring the investment cost, the entrepreneur chooses one of three possible projects – shirking, standard, or novel – which determines the probability with which the project will be in the good, middle, or bad state at $t = 1$. At Stage 3 $(t = 1)$, the state is observed. If state-contingent contracts are not allowed, the party holding control rights decides whether to liquidate the project at this date or to continue it to the next date; renegotiation is permitted. At Stage 4 $(t = 2)$, the returns are realized. We look for the pure strategy subgame-perfect Nash equilibrium and solve for it using backwards induction.

3. State-contingent contracts

In this section, we assume that at $t = 1$ the state is observable and verifiable, and hence, contractible. Thus, the contract specifies the promised repayment in each state – $R_{0,2} = R$ in the good state, $R_{0,2} = R_M$ in the middle state, and the fraction of the liquidation proceeds that the entrepreneur obtains in the bad state, μ. In the implementation of the first-best outcome, there are three relevant constraints which we illustrate below.

The first incentive constraint is that the entrepreneur must obtain a greater payoff from choosing the novel project compared to shirking:

$$\beta_B \mu L + \beta_G q_G (Y - R) - c \geq \mu L + b$$

$$\implies R \leq Y - \frac{\beta_G \mu L + (b + c)}{\beta_G q_G} \equiv R_{IC_1} \qquad (IC_1)$$

Totally differentiating IC_1 with respect to R and μ:

$$\left.\frac{dR}{d\mu}\right|_{IC_1} = -\frac{L}{q_G} < 0 \qquad (1)$$

The IC_1 constraint is linear in the (R, μ) space and its slope is negative.

The second incentive constraint is that the entrepreneur must obtain a greater payoff from choosing the novel project compared to choosing the standard project:

$$\beta_B \mu L + \beta_G q_G (Y - R) - c \geq \alpha_M q_M (Y - R_M) + \alpha_G q_G (Y - R) \qquad (IC_2)$$

This constraint is relaxed to the maximum extent possible by setting a high repayment in the middle state, $R_M = Y$, without affecting other constraints. We re-write the IC_2 constraint as follows:

$$R \leq Y - \frac{c - \beta_B \mu L}{(\beta_G - \alpha_G) q_G} \equiv R_{IC_2} \qquad (2)$$

Totally differentiating IC_2 with respect to R and μ:

$$\left.\frac{dR}{d\mu}\right|_{IC_2} = \frac{\beta_B L}{(\beta_G - \alpha_G)q_G} > 0 \tag{3}$$

The IC_2 constraint is linear in the (R, μ) space and its slope is positive. Since IC_1 is downward-sloping and IC_2 is upward-sloping, they necessarily intersect. The intersection occurs at:

$$R = Y - \frac{b\beta_B + c}{\alpha_M \beta_G q_G} \equiv R_A \tag{4}$$

$$\mu = \frac{\alpha_G(b+c) - b\beta_G}{\alpha_M \beta_G L} \equiv \mu_A \tag{5}$$

We say that the novel project is unlikely to fail if:

$$\beta_B < 1 - \alpha_G\left(\frac{c}{b} + 1\right) \equiv \hat{\beta} \tag{6}$$

From Eqs. (4) and (6) we can have the following lemma:

Lemma 1. *$\mu_A < 0$ if and only if the probability of failure of the novel project is sufficiently low, i.e., when $\beta_B < \hat{\beta}$. Conversely, if the novel project is likely to fail, $\beta_B \geq \hat{\beta}$, then $\mu_A \geq 0$.*

Finally, we turn to the investors' participation constraint:

$$\begin{aligned} &\beta_B(1-\mu)L + \beta_G q_G R - I \geq 0 \\ \implies &R \geq \frac{I - \beta_B(1-\mu)L}{\beta_G q_G} \equiv R_{PC} \end{aligned} \tag{PC}$$

The project is liquidated in the bad state, in which case investors obtain $(1-\mu)L$. In the good state, the project is continued to the next date and investors obtain $q_G R$. They provide the required funds, I, only if they at least break even in expectation. Totally differentiating PC with respect to R and μ:

$$\left.\frac{dR}{d\mu}\right|_{PC} = \frac{\beta_B L}{\beta_G q_G} > 0 \tag{7}$$

The PC constraint is linear in the (R, μ) space and its slope is positive.

The implementation of the novel project requires that the intersection between the sets of incentive compatible contracts and the set of contracts which satisfy the investors' participation constraint is not empty. We substitute (R_A, μ_A) into the PC constraint to obtain the condition on parameters such that the implementation of the novel project is possible for the case of $\mu_A \geq 0$:

$$\underbrace{\beta_G q_G Y + \beta_B L - I - c}_{S_{novel}} \geq b + \frac{\alpha_G(b+c) - b\beta_G}{\beta_G(1-\alpha_G)} = b + \mu_A L \tag{8}$$

Since $\mu \geq 0$, whenever $\mu_A < 0$, we set $\mu = 0$ and the relevant condition required for the implementation of the novel project becomes $S_{novel} \geq b$.

We summarize these results in the following proposition:

Proposition 1. *If $\mu_A \geq 0$, a necessary and sufficient condition for the implementation of the novel project (first-best) is $S_{novel} \geq b + \mu_A L$. If $\mu_A < 0$, the condition becomes $S_{novel} \geq b$.*

Below we consider the two cases, $\beta_B < \hat{\beta}$ and $\beta_B \geq \hat{\beta}$, with the latter further divided into two sub-cases. If the novel project is unlikely to fail, $\beta_B < \hat{\beta}$, the IC_1 constraint lies entirely below the IC_2 constraint in the admissible parameter range, $\mu \in [0, 1]$. This implies that if the entrepreneur chooses the novel project over shirking, it must be the case that she chooses the novel project over the standard project as well, so the IC_2 constraint can be ignored, and the necessary and sufficient condition for the implementation of the novel project is $S_{novel} \geq b$. If, by contrast, the novel project is likely to fail, $\beta_B \geq \hat{\beta}$, the IC_1 and IC_2 constraints intersect in the admissible parameter range. This implies that which IC constraint binds depends on the parameters. Then, the necessary and sufficient condition for the implementation of the novel project is given by Eq. (8). For the rest of the analysis, we assume that the implementation conditions are satisfied.

3.1. Case 1: $\beta_B < \hat{\beta}$

In this section, we consider the case that the novel project is unlikely to fail, $\beta_B < \hat{\beta}$. From Lemma 1, we know that in this case, the IC constraints intersect for some negative μ, i.e., $\mu_A < 0$. Then, the relevant constraints for implementation of the novel project are the PC and IC_1 constraints. The value of μ at which the PC and IC_1 constraints intersect is given as follows:

$$\mu = \frac{\beta_B L + \beta_G q_G Y - I - c - b}{L} \equiv \mu_B \tag{9}$$

$\mu_B > 0$ if $S_{novel} > b$, which is assumed to hold. We illustrate the three constraints for this case in Fig. 1. We illustrate the three constraints for this case in Fig. 1.

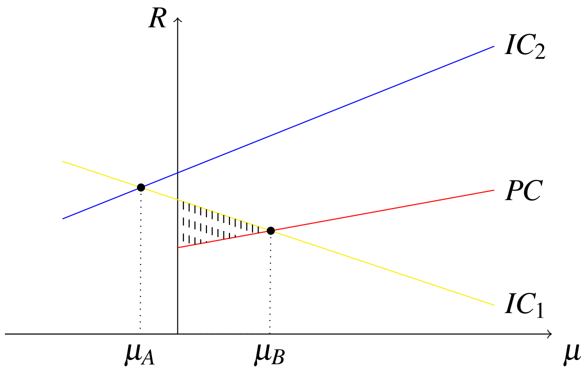

Fig. 1. State-contingent contracts, $\beta_B < \hat{\beta}$.

Lemma 2. *Suppose that $\beta_B < \hat{\beta}$. There exist a continuum of equilibria which all implement the novel project. Each equilibrium is characterized by a pair, (R, μ), where $\mu \in [0, \mu_B]$ and $R = R_{PC} = \frac{I-\beta_B(1-\mu)L}{\beta_G q_G}$. In all these pairs, investors make zero profits in expectation.*

Proof. We present a graphical proof and refer to Fig. 1. The shaded area represents the set of feasible contracts which can be used to implement the novel project since all these contracts satisfy both the IC_1 and IC_2 constraints and the PC constraint.

Suppose that a contract chosen is in the shaded region above the participation constraint – the entrepreneur picks the novel project. This cannot be an equilibrium because a competitive investor can deviate by offering an interest rate which is arbitrarily smaller, and thereby, profitably attract the borrower. Since the IC_1 and IC_2 constraints are slack in this region, this deviation does not affect the entrepreneur's incentives. This process iterates till the PC constraint binds, as per the standard Bertrand argument.

Consider the case that the offered contract is outside the shaded area. In this case, the novel project cannot be implemented because the offer will violate at least one of the constraints. An investor will profitably deviate by choosing a contract in the shaded region slightly above the participation constraint. By doing so, the investor will attract an entrepreneur who will implement the novel project, and the entrepreneur will extract almost the full surplus generated from it. Since the novel project is more profitable than the standard project or shirking, this deviation from the conjectured equilibrium makes the entrepreneur strictly better off, and hence, the conjectured equilibrium cannot sustain.

The entrepreneur is indifferent for any (μ, R) pair on the participation constraint for $\mu \in [0, \mu_B]$. To see this, we write down the entrepreneur's expected profit when investing in the novel project:

$$u_{novel} = \beta_B \mu L + \beta_G q_G (Y - R) - c \tag{10}$$

Totally differentiating u_{novel} with respect to R and μ, we derive its slope in the (R, μ) space:

$$\left.\frac{dR}{d\mu}\right|_{u_{novel}} = \frac{\beta_B L}{\beta_G q_G} \tag{11}$$

The slope is the same as that of the investors' participation constraint implying that there exists an indifference curve of the entrepreneur which coincides with the investors' participation constraint.[1] Therefore, the entrepreneur is indifferent among all points along the participation constraint, provided that they allow the novel project to be undertaken. ☐

3.2. *Case 2: $\beta_B \geq \hat{\beta}$*

In this section, we consider the case that the novel project is likely to fail, i.e., $\beta_B \geq \hat{\beta}$, implying that the IC constraints intersect at some positive μ (see Lemma 1). The PC constraint intersects the vertical axis at a lower point than the intersection of the IC_2 constraint and the vertical axis, i.e., $R_{PC} \leq R_{IC_2}$ evaluated at $\mu = 0$ if:

$$S_{novel} \geq \frac{c\alpha_G}{\beta_G - \alpha_G} \tag{12}$$

Below we consider the two cases: $S_{novel} \geq \frac{c\alpha_G}{\beta_G-\alpha_G}$ and $S_{novel} < \frac{c\alpha_G}{\beta_G-\alpha_G}$. In the former (resp. latter) case, the PC constraint intersects the vertical axis at a lower (resp. higher) point than the intersection of the IC_2 constraint and the vertical axis. The value of μ at which the PC and IC_2 constraints intersect is given as follows:

$$\mu = \frac{(\beta_G - \alpha_G)(I - \beta_B L - \beta_G q_G Y) + \beta_G c}{\beta_B \alpha_G L} \equiv \mu_C \tag{13}$$

[1] There exists a family of indifference curves in the (R, μ) space parallel to the investors' participation constraint, with the lower curves representing a higher expected utility for entrepreneurs. Within the feasible (shaded) region, the highest such curve is reached when the IC_1 constraint intersects the vertical axis (for $\mu_A < 0$, see Fig. 1) or when the two IC constraints bind (for $\mu_A \geq 0$, see Figs. 2 and 3); this is the least favorable outcome for the entrepreneur. The lowest curve coincides with the investors' participation constraint (the most favorable outcome for the entrepreneur in the feasible region). With competitive investors, the equilibrium must lie on this curve, implying that the participation constraint binds.

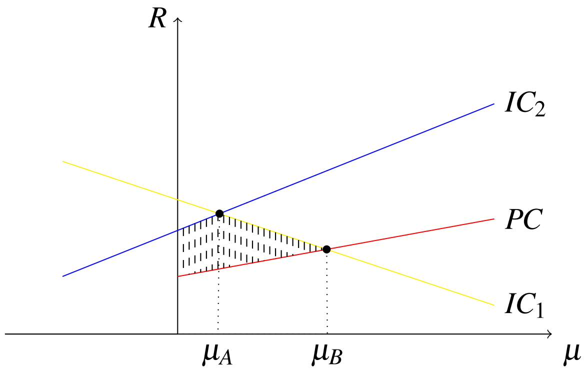

Fig. 2. State-contingent contracts, $\beta_B \geq \hat{\beta}$ and $S_{novel} \geq \frac{c\alpha_G}{\beta_G - \alpha_G}$.

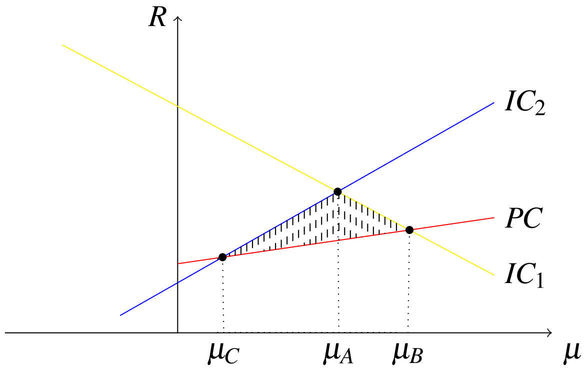

Fig. 3. State-contingent contracts, $\beta_B \geq \hat{\beta}$ and $S_{novel} < \frac{c\alpha_G}{\beta_G - \alpha_G}$.

$\mu_C < 0$ if $S_{novel} > \frac{c\alpha_G}{\beta_G - \alpha_G}$ (see Fig. 2) and $\mu_C > 0$ if $S_{novel} < \frac{c\alpha_G}{\beta_G - \alpha_G}$ (see Fig. 3).

Lemma 3. *Suppose that $\beta_B \geq \hat{\beta}$. There exist a continuum of equilibria which all implement the novel project. Each equilibrium is characterized by a pair, (R, μ), where $\mu \in [0, \mu_B]$ if $S_{novel} \geq \frac{c\alpha_G}{\beta_G - \alpha_G}$ and $\mu \in [\mu_C, \mu_B]$ if $S_{novel} < \frac{c\alpha_G}{\beta_G - \alpha_G}$, and $R = R_{PC} = \frac{I - \beta_B(1-\mu)L}{\beta_G q_G}$. In all these pairs, investors make zero profits in expectation.*

Proof. The proof uses Fig. 2 (corresponding to $S_{novel} \geq \frac{c\alpha_G}{\beta_G - \alpha_G}$) and Fig. 3 (corresponding to $S_{novel} < \frac{c\alpha_G}{\beta_G - \alpha_G}$) and follows the same arguments as in the proof of Lemma 2. The shaded area in each figure represents the set of feasible contracts which can implement the novel project. □

3.3. Comparison with Mitkov (2024)

Proposition 1 in Mitkov (2024) suggests that the equilibrium is characterized as follows: If $\beta_B < \hat{\beta}$, then the entrepreneur obtains 0 in the bad and middle states and $\frac{b+c}{\beta_G q_G}$ in the good state. In our analysis, this corresponds to the point of the intersection of the IC_1 constraint and the vertical axis in Fig. 1 which gives us the promised repayment to investors in the good state, $R = Y - \frac{b+c}{\beta_G q_G}$, verifying that the entrepreneur's payoff in this state is $Y - R = \frac{b+c}{\beta_G q_G}$. If $\beta_B \geq \hat{\beta}$, then the entrepreneur obtains $\frac{\alpha_G c - (\beta_G - \alpha_G) b}{\alpha_M \beta_G}$ in the bad state, and 0 in the middle state, and $\frac{c + \beta_B b}{\alpha_M \beta_G q_G}$ in the good state. In our analysis, this corresponds to the point of the intersection of the IC_1 and IC_2 constraints, (R_A, μ_A), in Figs. 2 and 3. So, the entrepreneur obtains $Y - R_A$ in the good state, 0 in the middle state, and $\mu_A L$ in the bad state. We verify $Y - R_A = \frac{c + \beta_B b}{\alpha_M \beta_G q_G}$ and $\mu_A L = \frac{\alpha_G c - (\beta_G - \alpha_G) b}{\alpha_M \beta_G}$.

We derive the conditions under which the state-contingent contract can achieve the first-best outcome (see Proposition 1). Our approach identifies the conditions under which pay-for-failure is necessary: $\beta_B \geq \hat{\beta}$ and $S_{novel} < \frac{c\alpha_G}{\beta_G - \alpha_G}$ (see Fig. 3). If $\beta_B \geq \hat{\beta}$ and $S_{novel} \geq \frac{c\alpha_G}{\beta_G - \alpha_G}$ (see Fig. 2), Mitkov (2024) argues that pay-for-failure is required (i.e., a strictly positive μ must be chosen) to implement the novel project. In contrast, we show that the novel project can be implemented even with $\mu = 0$.

4. Non-contingent contracts

In this section, we assume that the project's state at $t = 1$ is observable but not verifiable, implying that it is not feasible to write state-contingent contracts (implying that μ is exogenous). Mitkov (2024) considers if the choice of debt maturity can induce investment in the innovative project. Specifically, he considers short-term debt and callable long-term debt, which we analyze below.

4.1. Short-term debt

With short-term debt, the entrepreneur borrows I at $t = 0$ by issuing debt with face value, R_{01}, which matures at $t = 1$. At $t = 1$, the entrepreneur can either repay the debt or roll it over by promising up to $q_i Y$ (since the state is observable). We set $R_{01} > q_M Y$ (see footnote 2). Then, in the bad or the middle state, the entrepreneur cannot avoid default, and investors obtain the right to liquidate and take L (in the bad state), or roll-over and set the repayment rate in the middle state (say $R_{12}(M)$) equal to Y. In the good state, new debt is issued with face value R_{12}. The investors' participation constraint becomes:

$$\beta_G R_{01} + \beta_B L \geq I \tag{14}$$

Substituting $R_{01} = q_G R_{12}$, we derive the minimum face value of the rolled over debt:

$$R_{12} \geq \frac{I - \beta_B L}{\beta_G q_G} \equiv R_{12,PC} \tag{15}$$

The entrepreneur chooses the novel project over shirking if:

$$\begin{aligned} &\beta_G q_G (Y - R_{12}) - c \geq b \\ \Longrightarrow\ & R_{12} \leq Y - \frac{b + c}{\beta_G q_G} \equiv R_{12,IC_1} \end{aligned} \tag{16}$$

The entrepreneur chooses the novel project over the standard project if:

$$\begin{aligned} &\beta_G q_G (Y - R_{12}) - c \geq \alpha_G q_G (Y - R_{12}) + \underbrace{\alpha_M q_M (Y - R_{12}(M))}_{=0 \text{ since } R_{12}(M)=Y} \\ \Longrightarrow\ & R_{12} \leq Y - \frac{c}{(\beta_G - \alpha_G) q_G} \equiv R_{12,IC_2} \end{aligned} \tag{17}$$

Below, we assume that the conditions for implementing the novel project, as stated in Proposition 1, are satisfied. Suppose that the novel project is not very likely to fail, $\beta_B < \hat{\beta}$. Then, we obtain the following: $R_{12,PC} < R_{12,IC_1} < R_{12,IC_2}$. This implies that for $\beta_B < \hat{\beta}$, it is always possible to induce the entrepreneur to invest in the novel project while investors break even in expectation. Competitive forces ensure that the investors' expected profits are zero in equilibrium (i.e., their participation constraint binds).

Suppose that the novel project is likely to fail, $\beta_B \geq \hat{\beta}$. Now we have $R_{12,IC_1} > R_{12,IC_2}$. It remains the case that $R_{12,IC_1} > R_{12,PC}$, i.e., the IC_1 constraint never binds. The relation between $R_{12,PC}$ and R_{12,IC_2} is not clear. It is feasible to use short-term debt to induce investment in the novel project when $R_{12,PC} \leq R_{12,IC_2}$, which is the case if Eq. (12) is satisfied, i.e., $S_{novel} \geq \frac{c\alpha_G}{\beta_G - \alpha_G}$. Thus, when $\beta_B \geq \hat{\beta}$, it is possible for short-term debt to induce the entrepreneur to invest in the novel project while investors break even in expectation for when S_{novel} is sufficiently large. Competitive forces ensure that the investor's participation constraint binds, i.e., $R_{12} = R_{12,PC}$.[2] However, for $S_{novel} < \frac{c\alpha_G}{\beta_G - \alpha_G}$, the minimum interest rate on the rolled over debt which allows investors to break even in expectation exceeds the maximum interest rate such that the entrepreneur prefers the novel project over taking the standard project.

Proposition 2. *Short-term debt induces investment in the novel project when either i. $\beta_B < \hat{\beta}$ or ii. $\beta_B \geq \hat{\beta}$ and $S_{novel} \geq \frac{c\alpha_G}{\beta_G - \alpha_G}$. The investors' participation constraint binds, $R_{01} = q_G R_{12,PC}$. When $\beta_B \geq \hat{\beta}$ and $S_{novel} < \frac{c\alpha_G}{\beta_G - \alpha_G}$, short-term debt cannot induce investment in the novel project.*

Since for $\beta_B \geq \hat{\beta}$ and $S_{novel} < \frac{c\alpha_G}{\beta_G - \alpha_G}$, short-term debt does not induce investment in the novel project, we consider below if long-term debt can do so.

4.2. Long-term debt

With long-term debt, the entrepreneur borrows I at $t = 0$ with face value $R_{02} > L$. If the bad state is realized at $t = 1$, it will be efficient to liquidate the project, but unlike in the case of short-term debt, the investor cannot unilaterally do so since the entrepreneur is not in default. In this case, there is renegotiation in which the surplus from liquidating, $L - q_B Y$, is split according to the bargaining power μ. Additionally, the debt must be callable to preserve the entrepreneur's incentives to invest in the novel project. Specifically, the entrepreneur can call the debt at $t = 1$ with the repayment of a pre-specified amount, R_{01}. Re-financing at $t = 1$ is feasible only if $q_i Y > R_{01}$ since investors can observe the state. Moreover, the contract specifies that if the entrepreneur unilaterally liquidates

[2] We must verify that $R_{01} \equiv q_G R_{12,PC} > q_M Y$. This inequality is ensured by the parametric restriction $\beta_G q_M Y < I - \beta_B L$, which is the analogue of Equation (5) in Mitkov (2024).

the project at $t = 1$, she gets 0 while the liquidation proceeds go to the investors. This feature of the contract is consistent with the observation that debt-holders possess the highest priority in the case of liquidation, and in the context of the model, it ensures that the entrepreneur does not liquidate the project when it is inefficient to do so. The investors' participation constraint becomes:

$$\beta_B(q_B R_{02} + (1-\mu)(L - q_B Y)) + \beta_G(\min(\min(R_{01}, q_G Y), q_G R_{02})) \geq I \tag{18}$$

In the optimal callable debt contract, we set $R_{02} = Y$ and $q_M R_{02} < R_{01} < q_G R_{02}$. The lower bound on R_{01} ensures that it is too costly for the entrepreneur to call the debt back at $t = 1$ if the middle state has realized since refinancing is too costly. The upper bound on R_{01} ensures that in the good state, calling the debt back by refinancing is cheaper for the entrepreneur than not calling the debt back. Thus, the bounds on R_{01} ensure that the entrepreneur calls the debt if and only if the project is in the good state. We re-write Eq. (18) as follows:

$$\beta_B(q_B Y + (1-\mu)(L - q_B Y)) + \beta_G R_{01} \geq I$$
$$\Longrightarrow R_{01} \geq \frac{1}{\beta_G}[I - \beta_B L + \beta_B \mu(L - q_B Y)] \equiv R_{PC'} \tag{19}$$

The entrepreneur prefers the novel project over shirking if:

$$q_G Y - R_{01} \geq q_B(Y - R_{02}) + \mu(L - q_B Y) + \frac{b+c}{\beta_G} \tag{20}$$

With $R_{02} = Y$, we re-write Eq. (20) as follows:

$$R_{01} \leq q_G Y - \frac{b+c}{\beta_G} - \mu(L - q_B Y) \equiv R_{IC'_1} \tag{IC'_1}$$

Next we compare the payoffs from the novel and standard projects. To enable this comparison, note that the entrepreneur's payoff in the middle state is 0. This is true because the entrepreneur can neither unilaterally liquidate the project, nor do investors have an incentive to renegotiate the original contract in the middle state. Thus, the middle state can be ignored in equilibrium analysis. The entrepreneur prefers the novel project over the standard project if:

$$q_G Y - R_{01} \geq \frac{c - \beta_B \mu(L - q_B Y)}{\beta_G - \alpha_G}$$
$$\Longrightarrow R_{01} \leq q_G Y - \frac{c - \beta_B \mu(L - q_B Y)}{\beta_G - \alpha_G} \equiv R_{IC'_2} \tag{IC'_2}$$

We derive the intersections of the various constraints, μ'_A, μ'_B, and μ'_C as follows:

$$R_{IC'_1} = R_{IC'_2} \implies \mu = \frac{\alpha_G(b+c) - b\beta_G}{\alpha_M \beta_G(L - q_B Y)} \equiv \mu'_A = \frac{\mu_A L}{L - q_B Y} \tag{21}$$

$$R_{PC'} = R_{IC'_1} \implies \mu = \frac{\beta_B L + \beta_G q_G Y - I - c - b}{L - q_B Y} \equiv \mu'_B = \frac{\mu_B L}{L - q_B Y} \tag{22}$$

$$R_{PC'} = R_{IC'_2} \implies \mu = \frac{(\beta_G - \alpha_G)(I - \beta_B L - \beta_G q_G Y) + \beta_G c}{\beta_B \alpha_G(L - q_B Y)} \equiv \mu'_C = \frac{\mu_C L}{L - q_B Y} \tag{23}$$

When considering long-term debt, in Figs. 1–3, we replace the labels R, μ_A, μ_B, and μ_C with R_{01}, μ'_A, μ'_B, and μ'_C, respectively. When the shaded region in a figure (representing the set of feasible equilibria) touches the vertical axis, short-term debt (which sets $\mu = 0$) can implement the novel action. When the shaded region does not intersect the vertical axis, only callable long-term debt can induce investment in the novel project. We summarize these results below:

Proposition 3. *Suppose that either i. $\beta_B < \hat{\beta}$ or ii. $\beta_B \geq \hat{\beta}$ and $S_{novel} > \frac{c\alpha_G}{\beta_G - \alpha_G}$. In this case, long-term debt implements the novel project for μ sufficiently small, $\mu \in [0, \mu'_B]$. Suppose that $\beta_B \geq \hat{\beta}$ and $S_{novel} < \frac{c\alpha_G}{\beta_G - \alpha_G}$. In this case, long-term debt implements the novel project for intermediate values of μ, $\mu \in [\mu'_C, \mu'_B]$. In all cases in which financing is feasible, the investors' participation constraint binds, $R_{01} = R_{PC'}$.*

While the constraints for long-term debt align similarly with those under state-contingent contracts, the key difference lies in renegotiation. In state-contingent contracts, the split of liquidation proceeds in the bad state is determined endogenously, i.e., μ is contractible. By contrast, in the case of long-term debt, the split is governed by an exogenous bargaining protocol.

Corollary 1. *Suppose that $\beta_B \geq \hat{\beta}$ and $S_{novel} < \frac{c\alpha_G}{\beta_G - \alpha_G}$. For these parameters, pay-for-failure is necessary to induce investment in the novel project. In this case, long-term callable debt can replicate the state-contingent benchmark for a strictly positive measure of parameters, $\mu \in [\mu'_C, \mu'_B]$.*

Importantly, Mitkov (2024) argues that callable long-term debt implements the novel project only in knife-edge cases such as when the exogenous bargaining power parameter μ exactly coincides with the point where the two IC constraints intersect. Hence, under Mitkov (2024)'s reasoning, long-term debt would almost never replicate the state-contingent benchmark in practice, implying that long-term debt will be observed with zero probability. Instead, we show that there exists a full range with a strictly positive mass, $\mu \in [\mu'_C, \mu'_B]$, for which callable long-term debt can implement the novel project.

4.3. Choice of maturity

Suppose that either (i) $\beta_B < \hat{\beta}$ (see Fig. 1), or (ii) $\beta_B \geq \hat{\beta}$ and $S_{novel} \geq \frac{c\alpha_G}{\beta_G - \alpha_G}$ (see Fig. 2). In these cases, there is no need for pay-for-failure, and hence, short-term debt (which sets $\mu = 0$) can induce investment in the novel project. Long-term debt can also induce investment in the novel project for $\mu \in [0, \mu'_B]$. Note that in case (ii), Mitkov (2024) argues that pay-for-failure is necessary to induce investment in the novel project and concludes that the state-contingent outcome can be replicated only in knife-edge cases. Instead, we show that short-term debt can always induce the novel project in these cases. The implication is that, even when the novel project meets these conditions, state-contingent outcomes are feasible without resorting to knife-edge parameter values.

When (iii) $\beta_B \geq \hat{\beta}$ and $S_{novel} < \frac{c\alpha_G}{\beta_G - \alpha_G}$ (see Fig. 3), it is necessary to pay the entrepreneur for failure to induce investment in the novel project, and hence, short-term debt is ineffective. In this case, long-term debt can induce investment in the novel project for intermediate values of μ, $\mu \in [\mu'_C, \mu'_B]$. Thus, in contrast to Mitkov (2024) which predicts that callable long-term debt will be observed with zero probability, our analysis predicts that callable long-term debt will be observed with a strictly positive probability.

5. Conclusion

Mitkov (2024) considers whether callable long-term debt can induce investment in innovative projects when pay-for-failure is necessary. Restricting attention to the least-cost contract, at least one of the two incentive constraints faced by the entrepreneur binds. Callable long-term debt then implements the novel project for only a single value (a set of measure zero) of the exogenous bargaining power parameter, μ. We revisit this model and generalize its findings by considering any contract which satisfies the investors' zero profit condition, of which the least-cost contract (with ex ante transfers) is a special case. We show that equilibria exist in which both incentive constraints are simultaneously slack, so that callable long-term debt implements the novel project for a strictly positive measure of values of the exogenous bargaining power parameter, rather than at knife-edge parameters alone. Our results thus extend the results in Mitkov (2024) and provide a rationale for the use of callable long-term debt in this setting.

CRediT authorship contribution statement

Sonny Biswas: Writing – review & editing, Writing – original draft, Formal analysis, Conceptualization; **Kostas Koufopoulos:** Writing – review & editing, Writing – original draft, Formal analysis, Conceptualization; **Thi Mai Nguyen:** Writing – review & editing, Writing – original draft, Formal analysis, Conceptualization.

Data availability

No data was used for the research described in the article.

Declaration of competing interest

We have nothing to declare.

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