Efficient on-balance sheet funding with cross-subsidization across projects

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Abstract

A bank seeks funding for two projects at two different points in time but cannot commit to borrowing from the same investor in both periods. Despite this restriction, on-balance sheet funding with cross-subsidization across projects arises as the unique equilibrium and the outcome is efficient (second-best). Given the informational frictions considered, our solution strictly dominates off-balance sheet funding with voluntary support for all parameters, implying that a rational profitmaximizing bank will never choose the latter. We uncover two novel channels through which on-balance sheet funding can create value, which provides a rationale for the recent rise of Private Debt funds.

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1 Introduction

A bank has the choice to finance lending either on- or off- balance sheet. Segura and Zeng (2020) present a model to provide an economic rationale driven by information frictions for the emergence of off-balance sheet funding. Although they obtain an inefficient partial pooling equilibrium, they claim that off-balance sheet funding with voluntary support is a more effective signalling device than on-balance sheet funding. We reconsider their model, and show that a fully separating equilibrium arises at zero cost under on-balance sheet funding, and this solution strictly dominates the off-balance sheet funding solution for all parameter values. Furthermore, our analysis uncovers a second novel channel through which banks can create value: cross-subsidization across projects which relaxes an effort moral hazard constraint and boosts effort provision. By highlighting the advantages of the on-balance sheet funding mode, we provide a new explanation for the emergence and rise of the Private Debt funds in recent years.

In the model, there are two investment opportunities for a bank – the first suffers from an effort moral hazard problem, while the second suffers from an adverse selection problem. The bank pre-sells the expected net present value (NPV) of the adverse selection project and uses these funds to reduce the face value of debt which is a claim on the cash flow generated by the moral hazard project. The lower promised repayment relaxes the bank's effort moral hazard constraint and boosts effort provision in the first project. Regarding the project associated with adverse selection, we show there always exists a unique fully separating equilibrium in which only good banks invest. Separation arises through the violation of the bad bank's participation constraint rather than through the standard incentive compatibility constraint. We show that this equilibrium allocation coincides with the planner's solution (i.e., the equilibrium allocation is second-best).

Considering identical informational frictions, Segura and Zeng (2020) suggest that offbalance sheet funding allows a good bank to (partially) separate through the provision of voluntary support but it leads to a lower effort level in the first project. Thus, while the provision of voluntary support relaxes the adverse selection constraint in the second project, it makes the moral hazard constraint in the first project more binding. In contrast, in our solution, the funds raised from the pre-sale of the expected NPV of the adverse selection project are directly invested in the moral hazard project which relaxes the moral hazard constraint. Our separating equilibrium is costlessly (e.g., there is no money-burning) achieved through the violation of the participation constraint of the bad banks. Thus, while in the off-balance sheet funding mode the mitigation of the adverse selection problem has a perverse impact on the moral hazard constraint, in our case, the costless solution of the adverse selection problem frees up resources which are used to relax the moral hazard constraint. In summary, the off-balance sheet solution of Segura and Zeng (2020) is inefficient for three reasons: First, in their solution, good banks signal by burning money which leads to an inefficient partial pooling equilibrium in which some value-destroying bad banks invest. Second, since some bad banks (those who do not mimic the good banks) do not use the cash flows generated by the asset-in-place to repay the debt on the moral hazard project, the face value of debt for this project is higher which makes the effort moral hazard problem more binding and leads to lower effort provision. In contrast, signaling in our solution is not socially costly, and we show that there exists a unique equilibrium which is always fully separating where only good banks invest. Third, our solution increases effort provision even further by transferring the expected NPV of the second project to reduce the promised repayment for the first project. Hence, given the informational frictions in this model, the off-balance sheet funding mode is inefficient and does not maximize bank profit. Therefore, a rational profit-maximizing bank will never choose it.

In order to highlight the generality of our solution, we restrict attention to the case that the bank cannot commit to dealing with the same investor in both periods. This restriction implies that the contracts used must be zero profit on a contract-by-contract basis which makes the implementation of the second best potentially more difficult. Of course, the second-best outcome could also be implemented when the funds for both projects are offered by the same investor.¹

We show that the use of a part of the funds raised for one project for investment in a different project can relax informational constraints and create value for the bank. There is substantial evidence on the existence of cross-subsidization across different activities in financial institutions in various contexts (see e.g., Drucker and Puri 2005, Griffin et al. 2007, Santikian 2014, and Jenkinson et al. 2018). Our analysis draws attention to a different kind of cross-subsidization which has not been directly tested yet.

One potential application of our theory is to understand the recent trends in bank lending patterns which is that informationally sensitive credit is moving away from the traditional banks towards the Private Debt/Credit (PD) funds. In the last couple of decades, potentially due to binding regulatory constraints, traditional banks sell large portions of their loans in the secondary loan market or move them off their balance sheets through securitization. The evolution of the banking business away from on-balance sheet funding has coincided with the emergence of the PD funds, which is already a \$1.7 trillion market in 2023 and is rapidly growing. These PD funds make direct loans which are informationally sensitive and typically retain the entire loan on their balance sheets (see Block et al. 2024 and Haque et al. 2024). What explains the flight of informationally sensitive credit from the traditional banks to the PD funds? Our analysis implies that the

¹The solution for this case is available upon request.

on-balance sheet funding mode of PD funds is more effective at resolving informational frictions than the off-balance sheet funding mode of traditional banks. Thus, we present a new potential explanation for the emergence and rise of the PD funds in recent years.

Our theory generates two novel empirical predictions: First, the spread on debt is lower and monitoring intensity is higher for lenders with better growth prospects. Second, the spread on debt falls in the lender's growth opportunities if these are positively correlated with its core activities, while the direction of the effect is not clear if there is a negative correlation.

Relation to the wider literature.

In most existing theories, financial intermediaries have special skills in acquiring or processing information.² For example, Diamond (1984) and Holmstrom and Tirole (1997) emphasize the role of banks as monitors – in Diamond (1984) banks mitigate an ex post moral hazard problem through monitoring, while in Holmstrom and Tirole (1997) they mitigate an ex ante moral hazard problem. Ramakrishnan and Thakor (1984) and Allen (1990) focus on the screening role of banks in which banks can distinguish between good and bad borrowers. In these models, banks have a monitoring or screening cost advantage over non-intermediated outcomes (depending on the model, this advantage is either assumed, or it arises as an equilibrium outcome). In contrast, banks in our model do not have superior skills in resolving any informational frictions, yet they generate value through two novel channels:

First, in our model an efficient separating equilibrium arises through socially costless signaling, whereas, in existing banking theories, separation is achieved through costly information production. Second, in our model there is cross-subsidization across projects which is absent in existing models: the expected NPV of the adverse selection problem is transferred to the first project to relax the moral hazard constraint. Coval and Thakor (2005) also do not assume that banks have special information processing skills; rational banks form a beliefs-bridge between optimistic entrepreneurs and pessimistic investors. In our theory, all agents are rational, so we explore a different role of banks than the one considered by Coval and Thakor (2005).

Improved effort incentives due to cross-subsidization across projects is not a new idea. In Tirole (2006), cross-pledging across imperfectly correlated projects subject to effort moral hazard leads to greater effort provision by the manager (see also Diamond 1984, Cerasi and Daltung 2000, Laux 2001, Axelson et al. 2009, and Maurin et al. 2023) – the optimal contract entails that the bank is only compensated when all projects succeed; this relaxes the limited liability constraint of the agent and induces the efficient level

²There is a separate strand of the literature which focuses on the liquidity creation/provision role of banks (see e.g., Diamond and Dybvig 1983, Gorton and Penacchi 1990, Diamond and Rajan 2001, and Donaldson et al. 2018).

of effort provision. The cross-pledging mechanism relies on exploiting the diversification effect resulting from imperfectly correlated projects and it is ineffective if projects are perfectly correlated. Our mechanism works differently: There exists a unique separating equilibrium in which only good banks invest and they effectively pre-sell the expected NPV of the second project. They invest the proceeds from this sale in the first project which reduces the amount of external funding required for this project. This allows the bank to retain a higher fraction of the cash flow generated by this project, and therefore, it exerts more effort. Thus, in contrast to existing models, the correlation structure across projects does not play a role which implies that the cross-subsidization channel in our model is distinct from the well-known diversification effect.

2 Model

2.1 Set-up

There are four dates $t \in \{0, 1, 2, 3\}$. There is a bank which has no funds of its own at t = 0 and obtains a pay-off from assets-in-place at t = 1. The bank has two investment opportunities, the first at t = 0 and the second at t = 2; each project is of scale 1 unit and generates a payoff in the period following the investment. The funds are raised from competitive external investors. All agents are risk-neutral and the discount rate is 0.

The first project is always good (g), while the second project may be either good or bad (b). In either case, a project produces R per unit of investment if it succeeds and zero if it fails. A project succeeds with probability p_i . We assume that only good projects are profitable:

A1: $p_g R > 1 > p_b R$

The first project to be undertaken at t = 0 is subject to effort moral hazard. The project succeeds with probability p_g and fails with probability $1 - p_g$. The bank can exert unobservable effort $e \in [0, 1]$ at a cost c(e) to reduce the probability of failure. Taking into account the bank's effort provision, the first project's success probability is $p_g + me$ and its failure probability is $1 - p_g - me$, with $m \leq 1 - p_g$ to ensure non-negative probability of failure. m is a constant which is interpreted as the marginal value of effort. The cost of effort c(e) satisfies:

A2: c(0) = 0, c'(0) = 0, c'(e) > 0 and c''(e) > 0 for all e > 0, and c'(1) > mR

The bank obtains pay-off Y at t = 1 from its assets-in-place. We assume that Y is not so large such that the t = 0 debt becomes riskless, as otherwise the first-best effort level could be implementable: **A3:** $Y < 1 - \alpha(p_g R - 1)$

At t = 1, the bank privately learns whether the second project is good or bad. The second project to be undertaken at t = 2 is of type g with probability $\alpha \in (0, 1)$. The bank privately observes the project's type. We assume that α is small such that the average NPV of the pool is negative:

A4:
$$\alpha < \frac{1-p_b R}{(p_g - p_b)R}$$

The timeline is illustrated in Figure 1.

t = 0	t = 1	t = 2	t = 3
The bank raises funding for the 1st project.	Assets-in-place generate cash flows, Y.	Bank seeks remaining funding for the 2nd project	2nd project returns realize
It also pre-sells a fraction of the 2nd project.	1st project returns realize.	Investors provide funds if they believe project is good	l.
Then, it chooses effort level.	Bank privately learns second project's type.		



2.2 The Game

We consider a two-stage sequential game, the second stage of which itself is a standard two-stage signaling game.

• Stage 1: At t = 0 (when the bank is unaware of its own type), competitive investors propose the following contract: (D₁, D_{0,3}, K, Z).³ For a price, Z, the investor obtains debt with face value, D_{0,3}, which is a claim on the cash flow generated by the adverse selection project, if undertaken, and this debt is senior to any security which may be subsequently issued. The bank may use these funds to directly invest in the moral hazard project or consume it. If the bank chooses to invest it in the project (resp. consume it), it needs to raise an amount, 1 – Z units (resp. 1 unit), which the investor provides subject to the promised repayment, D₁, being sufficient for the investor to break even. The investor gives an amount, K, to the bank. The bank may use these funds to directly invest in the adverse selection project or consume it. In the next stage, the bank will try to raise funding from a potentially different investor playing a two-stage signaling game which follows.

 $^{^{3}}$ We obtain identical results if there are two separate contracts at this stage of the game offered by two different investors, one relating to the first project and another relating to the second project.

- Stage 2a (which occurs at t = 2): the bank, now privately aware of its type, seeks to raise funds for the second project from competitive investors (who may be different from the one who provided the funds at t = 0). The bank offers a contract which specifies whether K will be invested directly in the second project or not, and offers to investors debt with face value, $D_{2,3}$, which is junior to the debt issued at t = 0, in exchange for the remaining amount of funds necessary for undertaking the project.
- Stage 2b: given the contract offered by the bank in Stage 2a, investors will form beliefs about the second project's type. Given these beliefs, investors will decide whether they will accept the contract and provide the funds necessary to undertake the project or not.

We look for the pure strategy Perfect Bayesian equilibria of this game that satisfy the Intuitive Criterion of Cho and Kreps (1987).⁴ Note that we do not impose exogenously whether the bank pre-sells a fraction of the cash flows from the second project at t = 0 or not, i.e., K, Z, and $D_{0,3}$ are chosen optimally in equilibrium.

2.3 First-best

In the first-best, the effort level exerted by the bank is observable and verifiable, and the type of the second project is publicly known. In this case, the bank always undertakes the first project and, given Assumption 2, the level of effort is:

$$c'(e^{FB}) = mR \tag{1}$$

The bank invests in the second project only if it is good.

2.4 Equilibrium

The two key dates of strategic interaction between the bank and investors are t = 0 and t = 2 when the bank raises funds for the projects. We solve for the equilibrium using backward induction.

2.4.1 Funding at t = 2

Because the pair, $(K, D_{0,3})$, is determined at Stage 1 of the game, when the two-stage signalling game starts at t = 2 (Stage 2a), $(K, D_{0,3})$ are exogenously given. Hence, the

⁴The appropriate solution concept is Perfect Bayesian since in the second stage of the game banks may signal about their type through the use of K, which implies that inferences are made.

only endogenous variable to be chosen in Stage 2a is $D_{2,3}$. Given that, below we list all the candidate equilibria of the subgame which starts at t = 2.

- 1. A candidate separating equilibrium in which only bad banks obtain funding.
- 2. A candidate pooling equilibrium in which both firm types obtain funding.
- 3. A candidate pooling equilibrium in which neither firm type obtains funding (market breakdown).
- 4. A candidate separating equilibrium in which both good and bad banks obtain funding, but at different terms.
- 5. A candidate separating equilibrium in which only good banks obtain funding.

Below we consider each candidate equilibrium.

Lemma 1 There cannot exist a separating equilibrium in which only bad banks obtain funding or both good and bad banks obtain funding but at different terms.

Proof. The proof is in the Appendix.

The contract at Stage 1 specifies the pair, $(K, D_{0,3})$. Hence, at Stage 2a the bank can only choose $D_{2,3}$ to signal its type. Given that, bank *i*'s profit is denoted Π_i and its participation constraint is given by:

$$\Pi_i = p_i (R - (D_{0,3} + D_{2,3})) - K \ge 0 \tag{2}$$

We derive the slope of $\Pi_i = 0$ in the $(K, D_{2,3})$ space by totally differentiating Equation (2) with respect to $D_{2,3}$ and K:

$$\left. \frac{dD_{2,3}}{dK} \right|_{\Pi_i = 0} = -\frac{1}{p_i} < 0 \tag{3}$$

Investors at t = 2 facing a type *i* bank make an expected profit which is denoted γ_i , and they provide the necessary funds for investment if $\gamma_i \ge 0$:

$$\gamma_i = p_i \min(D_{2,3}, (R - D_{0,3})) - (1 - K) \ge 0 \tag{4}$$

In the main text, we consider the case that $D_{2,3} \leq R - D_{0,3}$ holds as this is the relevant case for the results.⁵ We derive the slope of $\gamma_i = 0$ in the $(K, D_{2,3})$ space by totally

⁵In Section 6.2, we consider the case of $D_{2,3} > R - D_{0,3}$.

differentiating Equation (4) with respect to $D_{2,3}$ and K:

$$\left. \frac{dD_{2,3}}{dK} \right|_{\gamma_i = 0} = -\frac{1}{p_i} < 0 \tag{5}$$

In Figure 2a, we plot the binding participation constraints of banks (red lines) and investors (blue lines) in the $(K, D_{2,3})$ space. Both $\Pi_i = 0$ and $\gamma_i = 0$ have a negative slope and are linear in the $(K, D_{2,3})$ space. Since the slopes are the same, $\Pi_i = 0$ and $\gamma_i = 0$ do not cross for any *i*. $\Pi_i = 0$ implies that $D_{2,3} = R - D_{0,3}$ for K = 0 and $D_{2,3} = R - D_{0,3} - \frac{1}{p_i}$ for K = 1. $\gamma_i = 0$ implies that $D_{2,3} = \frac{1}{p_i}$ for K = 0 and $D_{2,3} = 0$ for K = 1. A bank of type *i* is willing to undertake the project in the region below its participation constraint, while an investor facing a bank of type *i* is willing to provide funds in the region above the corresponding participation constraint. Additionally, we plot the pooling participation constraint (blue dashed line) – this is the average of γ_b and γ_g , and on the y-axis it lies above *R* since the average NPV of the pool is negative (Assumption *A4*).

From Equation (4), we derive the minimum promised repayment (face value of debt, $D_{2,3}$) for which investors at t = 2 will provide funds:

$$D_{2,3} \ge \frac{1-K}{p_i} \tag{6}$$

At this lower bound (i.e., when Equation (6) holds with equality), the intersections of $\Pi_i = 0$ and $\gamma_g = 0$ with the vertical axis coincide, and since $\Pi_g = 0$ and $\gamma_g = 0$ have the same slope, these two lines coincide (the red and blue line in Figure 2b). In this case, the shaded area in Figure 2a shrinks to the $\Pi_g = \gamma_g = 0$ line in Figure 2b.

Lemma 2 There cannot exist a pooling equilibrium in which both good and bad banks obtain funding.

Proof. The proof is in the Appendix.

Lemma 3 Depending on the terms of the contract offered in Stage 1 of the game, there either exists a pooling equilibrium in which neither bank obtains funding (market breakdown) or a separating equilibrium in which only good banks obtain funding.

Proof. We present a graphical proof and refer to Figure 2.

There exists a funding equilibrium in the shaded region in Figure 2a: here, investors' participation constraints are slack and they make positive profits when they face good banks (i.e., $\gamma_g > 0$), a good bank's participation constraint is satisfied (i.e., $\Pi_g \ge 0$),



Figure 2: Feasible equilibria

while a bad bank's participation constraint is (weakly) violated (i.e., $\Pi_b \leq 0$). Funding is infeasible above the shaded region since the good bank's participation constraint is violated. Below the shaded region, the market breaks down since either the bad bank's participation constraint is satisfied and/or investors' participation constraint is violated.

Both the breakdown equilibrium and the separating equilibrium with only good banks participating are feasible along the bad bank's participation constraint bordering the shaded region up to $\left(K = 0, D_{2,3} = \frac{1}{p_g}\right)$. For these parameters, a bad bank is indifferent between participating or not which implies that the breakdown equilibrium cannot be ruled out for these parameters in this subgame. Note in Figure 2b, when the lower bound on $D_{2,3}$ binds (see Equation (6)), the participation constraint of the bad bank lies entirely below the $\gamma_g = 0$ line and the coexistence of the breakdown equilibrium and the separating equilibrium is possible only at the vertical intercept, $\left(K = 0, D_{2,3} = \frac{1}{p_g}\right)$.

The proposition below summarizes the results from the Lemmas 1-3, and characterizes the equilibria that may arise in the two-stage signaling game played at t = 2:

Proposition 1 In the two-stage signaling game played at t = 2, there can exist two potential equilibria:

- 1. a separating equilibrium in which only good banks invest and
- 2. a pooling equilibrium in which no banks invest (market breakdown).

Below, we show that in the whole game the only equilibrium that survives is the separating equilibrium in which only good banks invest.

2.4.2 Funding at t = 0

At t = 0, investors offer a contract to the bank which specifies the terms and conditions under which they will purchase a fraction of the cash flow from the second project, $D_{0,3}$, at a price, Z, and also, provide the bank with the additional funds required for investment in the first project, 1 - Z. The lower bound on $D_{2,3}$ that we derived in the previous section (Equation (6)), puts an upper bound on how much of the cash flow from the second project can be sold to the t = 0 investors, and for a given K, this upper bound is as follows:

$$D_{0,3} \le R - \frac{1-K}{p_g} \tag{7}$$

If this upper bound is violated, there will be too little cash flow left over at t = 3 for the t = 2 investors, who will then refuse to provide the funds required to undertake the second project. If the first project succeeds, investors obtain the promised repayment, D_1 . In the case of failure, investors obtain Y - K. The bank retains K whether or not the first project succeeds. At t = 2, the bank privately knows the second project's type to be good or bad. For any pair, $(K, D_{0,3})$, which satisfies Equation (7), the t = 2 investors will offer the funds necessary for investment in the second project with face value of debt, $D_{2,3}$, conditional on the bank's decision of whether to invest K directly in the second project or to consume it. That is, conditional on the bank investing K directly, the pairs $(K, D_{2,3})$ that violate the bad bank's participation constraint while satisfying the good bank's one, and ensuring that investors make weakly positive profits (i.e., $\gamma_g \geq 0$) are those that belong to the shaded area in Figure 2. Given the contract offered, investors' zero profit condition at t = 0 is as follows:

$$\underbrace{(p_g + me)D_1 + (1 - p_g - me)Y - K - (1 - Z)}_{\text{Investors' expected profit from 1st project}} + \underbrace{\alpha p_g D_{0,3} - Z}_{\text{2nd project}} = 0$$
(8)

From the zero profit condition, we derive the repayment, D_1 . The bank's expected profits is:

$$\Pi_1 = \underbrace{(p_g + me)(R - D_1 + Y) - c(e) - Z}_{\text{Bank's expected profit from 1st project}} + \underbrace{\alpha p_g(R - (D_{0,3} + D_{2,3})) + (1 - \alpha)K + Z}_{\text{2nd project}}$$
(9)

The first part is the expected profit from the first project, net of the cost of effort. The second part is the expected profit from the second project depending on the bank's use of K. Given the contract chosen by the bank, it maximizes its expected profit from the first project by choosing the level of effort. Taking the first order condition with respect to the effort level, solving the investor's zero profit condition (Equation (8)) for D_1 , and

substituting it into the first order condition, we obtain the equilibrium effort level exerted by the bank:

$$c'(e^*) = m(R - D_1^* + Y)$$
(10)

$$= m\left(R - \frac{1-Y}{p_g + me} - \frac{K - \alpha p_g D_{0,3}}{p_g + me}\right)$$
(11)

Because c(e) is strictly convex in e, c'(e) is strictly increasing in e. Since the right hand side of Equation (11) is falling in K and increasing in $D_{0,3}$, a lower K and a higher $D_{0,3}$ implies a higher effort level. The intuition is as follows: The higher the face value of debt, $D_{0,3}$, the higher the amount of funds that the bank raises from pre-selling expected cash flows from the second project, which can then be invested in the first project. Hence, the remaining amount of funds to be raised to invest in the first project falls, and so the face value of debt issued at t = 0, D_1 , which is a claim on the cash flow generated by the first project falls. This, in turn, implies that the cash flow that the bank retains in the case that the first project succeeds is higher, which relaxes the effort moral hazard constraint, and therefore, the bank exerts more effort.

Below, we present two lemmas which allows us to characterize the equilibrium in Proposition 2.

Lemma 4 From the t = 0 perspective, there cannot exist an equilibrium in which a good project at t = 2 does not obtain funding.

Proof. The proof is in the Appendix.

Starting from a non-funding equilibrium, an investor can profitably deviate by offering a new pair, $(K, D_{0,3})$, which would allow the t = 2 investors to offer a $D_{2,3}$ in the shaded region in Figure 2a which satisfies the good bank's and investors' participation constraints and violates the bad bank's participation constraint at t = 2.

Lemma 5 There cannot exist an equilibrium in which the contract specifies K > 0. In any equilibrium, the contract must specify K = 0 and $D_{0,3} = R - \frac{1}{p_q}$.

Proof. The proof is in the Appendix.

Bank profit is maximized for the smallest possible K and the largest possible $D_{0,3}$ (from Equation (11)). Competition among investors drives K to 0 and $D_{0,3}$ to its upper bound, $R - \frac{1}{p_g}$. For any K > 0, there exists a deviation in which the t = 0 investors offer a smaller K and purchase a higher fraction of the cash flows from the second project, which, in turn, relaxes the bank's effort moral hazard constraint by reducing D_1 , and increases bank value. Effectively, a positive K implies that some resources are left unused at t = 0, which leads to an inefficiency. Note from Lemma 3, K = 0 can potentially lead to two equilibria in the second stage subgame: one is the separating one considered above and the other is the market breakdown equilibrium. But from the perspective of t = 0, the market breakdown equilibrium cannot exist due to the possibility of profitable deviations. Thus, the separating equilibrium with $\left(K = 0, D_{0,3} = R - \frac{1}{p_g}\right)$ is the unique equilibrium of the full game. While K equals 0 on the equilibrium path, it is the off-path threat of positive K that sustains the equilibrium.

Substituting the equilibrium values, $\left(K = 0, D_{0,3}^* = R - \frac{1}{p_g}\right)$ in Equation (8) we derive the promised repayment in the first project, D_1^* :

$$D_1^* = \frac{1 - (1 - p_g - m_e)Y}{p_g + m_e} - \frac{\alpha(p_g R - 1)}{p_g + m_e}$$
(12)

Lemmas 4 and 5 rule out all candidate equilibria, but one: $\left(D_1 = D_1^*, K = 0, D_{0,3}^* = R - \frac{1}{p_g}\right)$. Now we need to show that there is no profitable deviation from this equilibrium. To see why this is the case, note that any deviation would have to increase bank profits by relaxing the moral hazard constraint further. To do so, D_1 must be lower than D_1^* . However, since the full expected surplus from the second project is already used to minimize D_1 and investors are on their participation constraints, $D_1 < D_1^*$ is not feasible. Thus, this candidate equilibrium exists. Using $D_1 = D_1^*$, K = 0, and $D_{0,3}^* = R - \frac{1}{p_g}$ in Equation (11), the equilibrium effort level is given by:

$$c'(e^*) = m\left(R - \frac{1 - Y}{p_g + me} + \frac{\alpha(p_g R - 1)}{p_g + me}\right)$$
(13)

Given Assumption A3, $D_1^* > Y$, implying that the debt is risky. In turn, this prevents equilibrium level of effort from reaching the first-best level, i.e., $e^* < e^{FB}$. This is the reason why optimality requires that we set K = 0 (its lowest possible value) and $D_{0,3} = R - \frac{1}{p_g}$ (its highest value) in order to achieve the maximum possible effort level (second best, see Section 2.5). We characterize the equilibrium in Proposition 2.

Proposition 2 There exists a unique equilibrium in which the contract specifies $(D_1 = D_1^*, K = 0, D_{0,3}^* = R - \frac{1}{p_g})$ at t = 0. There is full separation in the second project and only good banks participate. Effort provision in the first project is given by $c'(e^*)$.

The contract in Proposition 2 does not include provision for funding the second project and competitive investors make zero profits in expectation, as in Segura and Zeng (2020). Still, our solution differs since the bank pre-sells the NPV of the second project at t = 0 to reduce the amount it needs to borrow for the first project. We do not impose how much of the cash flow from the second project the bank pre-sells, and allow for all possibilities, including no pre-selling (which corresponds to the solution in Segura and Zeng (2020)). We show that not pre-selling is off the equilibrium path. That is, a rational profitmaximizing bank will never choose to not pre-sell.

2.5 The planner's solution

The objective of the planner is to maximize the net social surplus (the bank value). The net social surplus consists of two parts: the value created by the project associated with adverse selection and the value created by the project associated with effort moral hazard. With regards to the project which is subject to adverse selection, maximization of the net social surplus requires that only good projects are undertaken. The planner can achieve this separation by giving K to the bank and allow banks to play a two-stage signaling game similar to the one we consider above (stages 2a and 2b) where the role of the investors is played by the planner. Furthermore, in order to maximize the aggregate net social surplus, the planner will extract the full expected NPV of the second project and transfer it to the first project to relax the effort moral hazard constraint as much as possible. Hence, at the optimum, he will set K = 0. Formally, the planner's problem reduces to:

$$\begin{array}{ll}
\text{Max} & (p_g + me)(R - \tau_1 + Y) - c(e) \\
\text{subject to} \\
\text{(IC)} & c'(e^*) = m(R - \tau_1 + Y) \\
\text{(PC)} & p_g(R - \tau_3) - K \ge 0 \ge p_b(R - \tau_3) - K \\
\text{(FC)} & (p_g + me)\tau_1 + (1 - p_g - me)Y - K + \alpha(p_g\tau_3 - (1 - K)) \ge 1 \\
\text{(LL)} & \tau_3 \le R
\end{array}$$
(14)

The planner provides the required funds and sets transfers from the bank to himself as τ_1 which is the repayment at t = 1 and τ_3 which is the repayment at t = 3. The planner maximizes the objective function with respect to the effort exerted by the bank subject to four constraints. The first constraint is the effort moral hazard constraint (IC). The second constraint is related to truth-telling about the second project. The third constraint is the planner's feasibility constraint (FC). The final constraint is the limited liability constraint.

The planner's objective is to maximize the net social surplus. Given that full separation can be achieved on the second project and the planner can extract the full surplus (NPV) on this project, the planner's problem reduces to maximizing the bank's effort in the first project. From the IC, this can be done by reducing τ_1 to the maximum extent possible (given Assumption A3, the first-best cannot be reached). To minimize τ_1 , the FC must bind. The choice variables are τ_3 and K. From the FC the aggregate effect of K is $-(1 - \alpha)K$, implying that a strictly positive K diverts resources which could be used to reduce τ_1 and increase effort. Thus, at the optimum, the planner will set K = 0. Also, from the second line of the FC, higher the τ_3 , lower is the τ_1 consistent with the planner's FC being satisfied. Thus, the planner will set the transfer to the maximum possible amount consistent with the limited liability constraint, i.e., $\tau_3 = R$. $\tau_3 = R$ and K = 0 satisfies the truth-telling constraint, and hence, is consistent with a separating equilibrium which allows the planner to extract the full surplus.⁶ Substituting K = 0 and $\tau_3 = R$ in the FC, we obtain $\tau_1 = D_1^*$ (see Equation (12)). Since $\tau_1 = D_1^*$, the IC implies that the effort level in the planner's solution coincides with that in the competitive equilibrium allocation (see Equation (10)), and hence, the competitive equilibrium allocation is optimal (second-best).

Proposition 3 The equilibrium allocation of the game coincides with the planner's solution, and hence, it is efficient (second-best).

Effectively, the planner lends to the bank at t = 0, and commits to lend again at t = 2, if the bank turns out to be of the high type. We show that the allocation in the planner's equilibrium can be replicated in the decentralized equilibrium even when we do not allow for the bank to form long-term relationships with fund providers.

3 Benchmarks

In this section, we contrast our solution with two benchmarks (market funding and offbalance sheet funding).

3.1 Market funding

Instead of the two projects being owned by the same bank, suppose that there are two separate banks, each managing one of the projects. We refer to this case as market funding since funds are raised separately for the two projects, leaving no scope for any interaction between them. We assume that the bank managing the first (resp. second) project has assets-in-place which produce $Y_1 > 0$ (resp. $Y_2 > 0$), with $Y_1 + Y_2 = Y$.

With regards to the second project, the good bank offers a pair (K, D_3) along the lower contour of the shaded region in Figure 2a (where D_3 corresponds to $D_{2,3}$, and $D_{0,3} = 0$). The offer is on the bad bank's participation constraint for $K \in [0, \bar{K}]$ and on

 $^{^{6}}$ By an argument similar to the one developed in Lemma 4 we can show that this is indeed the unique equilibrium in the game played by the planner and the investors.

investors' participation constraint (when facing good borrowers) for $K \in (\bar{K}, 1]$, where \bar{K} is the intersection point of Π_b and γ_g . It is assumed that bad banks stay out in case of indifference between investing or not. Despite perfect competition, investors obtain positive profits in expectation and the expected profits are smaller as direct investment by the bank increases (for $K > \bar{K}$, investors' expected profits become 0). In this case, the bank invests all available cash flows, $K = Y_2$, into the project to minimize sharing profits with investors. Regardless of the split of the surplus, the outcome is efficient since the equilibrium is separating and only good projects obtain funding for any $Y_2 > 0$.

With regards to the first project, the face value of debt is D_1 and the zero profit condition of investors is given as follows:

$$(p_g + me)D_1 + (1 - p_g - me)Y_1 - 1 = 0$$
(15)

From the zero profit condition, we derive the repayment, D_1^{D} . The firm maximizes its expected profit from the first project by choosing the level of effort:

$$\Pi_1^{\ D} = (p_g + me)(R - D_1^{\ D} + Y_1) - c(e)$$
(16)

Taking the first order condition, and substituting the investor's zero profit condition (Equation (15)), we obtain the equilibrium effort exerted:

$$D_1{}^D = \frac{1 - (1 - p_g - me)Y_1}{p_g + me}$$
(17)

$$c'(e^D) = m(R - D_1^D + Y_1)$$
(18)

Proposition 4 On-balance sheet bank funding strictly dominates market funding for all parameter values.

Proof. The proof is in the Appendix.

 $e^D < e^*$ for all parameter values. The reason is that with market funding, the two projects are separately financed and the borrower cannot bring forward the expected profits to investors in the second project to reduce the repayment for the first project, i.e., $D_1^D < D_1^*$ for any $Y_1 \leq Y$. Since the provider of effort retains more of the surplus from effort provision in the case of on-balance sheet bank funding, effort is higher in this case compared to the market financing case.

On-balance sheet bank funding and market funding are welfare-equivalent in the case of the second project, while bank funding delivers a strictly more efficient outcome in the case of the first project. The result is driven by cross-subsidization across projects in the case when the a single bank owns both projects.

3.2 On- versus Off-balance sheet

Segura and Zeng (2020) consider a very similar setting and consider the case that the first project may be funded on- or off-balance sheet. First, we recap their analysis in brief, and then we provide a comparison.

Under the on-balance sheet financing mode, investors in the first project have unlimited recourse to cash flows from the assets-in-place. In the absence of cross-subsidization across projects, the promised repayment to investors in the first project, D_1^{on} , and effort provision, e^{on} , are given by:

$$D_1^{on} = \frac{1 - (1 - p_g - me)Y}{p_g + me}$$
(19)

$$c'(e^{on}) = m(R - D_1^{on} + Y)$$
(20)

Under this funding mode, there is a partial pooling equilibrium in which all good banks and a fraction of bad banks invest in the second project.

Under the off-balance sheet financing mode, in the event of failure of the first project, there is no obligation for the bank to make repayments using cash flows arising from the bank's assets-in-place. However, the good bank can signal its second project's type by voluntarily using cash flows from the assets-in-place to repay the investors in the first project. The promised repayment to the investors in the first project, D_1^{off} , and effort provision, e^{off} , are given by:

$$D_1^{off} = \frac{1 - \alpha Y}{p_g + me} \tag{21}$$

$$c'(e^{off}) = m(R - D_1^{off})$$
(22)

The ratio of marginal cost of providing voluntary support to the marginal benefit is smaller for good banks compared to bad banks, implying that voluntary support can be an effective signalling device. There is a partial pooling equilibrium in which all good banks and a fraction of bad banks invest; due to the voluntary support, the fraction of bad banks investing in the off-balance sheet case is always smaller than in the on-balance sheet case.⁷

The trade-off between on-balance sheet financing (without cross-subsidization across projects) and off-balance sheet financing is the following: under on-balance sheet fi-

 $^{^{7}\}mathrm{If}$ cash flows from assets-in-place are large enough to make the second project riskless, there is full separation.

nancing, the effort provision in the first project is higher, while under off-balance sheet financing signalling in the second project is stronger. The optimal financing mode is determined from this trade-off. Our solution differs since we allow (but do not impose) the pre-sale of cash flows from the second project at t = 0. To facilitate comparisons, we present the following Lemma:

Lemma 6 The face value of debt and the corresponding effort levels under different financing modes are as follows: $D_1^* < D_1^{on} < D_1^{off}$ and $e^* > e^{on} > e^{off}$.

Proof. The proof is in the Appendix.

Our solution, (D_1^*, e^*) , strictly dominates both the on-balance sheet solution without cross-subsidization, (D_1^{on}, e^{on}) , and the off-balance sheet solution, (D_1^{off}, e^{off}) .

- 1. $e^* > e^{on} > e^{off}$ (Lemma 6). Effort provision in our solution dominates the effort provision in either case above for all parameter values since $D_1^* < D_1^{on} < D_1^{off}$. Intuitively, the pre-sale of the cash flows from the second project at t = 0 reduces the funding requirement of the first project, which reduces the promised repayment D_1 and, since the bank retains more of the surplus from exerting effort, effort provision is higher.
- 2. In relation to the second project, Segura and Zeng (2020) obtain a partial pooling equilibrium in which all good banks and a fraction of bad banks invest if cash flows from the assets-in-place are sufficiently high (inefficiency); otherwise, there is a market breakdown equilibrium with no funding at all (inefficiency). In contrast, our equilibrium is always separating in which only good banks invest (efficiency) even if the cash flows generated by the assets-in-place is 0.

Proposition 5 For all parameter values, on-balance sheet funding with cross-subsidization across projects strictly dominates on-balance sheet funding without cross-subsidization and off-balance sheet funding.

In the choice of the funding mode, Segura and Zeng (2020) trade-off the efficiency gains from mitigating the adverse selection constraint with the efficiency losses from tightening the moral hazard constraint; mitigating one makes the other constraint more binding. In our case, the existence of a fully separating equilibrium in the adverse selection project allows the transfer of the expected NPV of this project to the moral hazard project and this relaxes the moral hazard constraint; indeed, as we show in Section 2.5, our solution implements the second-best outcome. Hence, given the informational frictions in this model, the off-balance sheet funding mode is inefficient and does not maximize bank profit. Therefore, a rational profit-maximizing bank will never choose it.

4 Empirical implications

Our analysis implies that informationally sensitive loans should be financed on the balance sheet. Consistent with this prediction, in recent years, vast amounts of informationally sensitive lending has migrated away from banks, who increasingly use the off-balance sheet funding mode and sell their loans upon origination, towards Private Debt funds, who typically retain the entire loan that they issue on their balance sheets (see Block et al. 2024 and Haque et al. 2024). Viewed through the lenses of our model, the funding mode of PD funds which is the retention of loans on their balance sheets, confers upon them an advantage in making informationally sensitive loans to borrowers. We also derive two new testable empirical predictions, which we state below:

1. The spread on debt is lower and monitoring intensity is higher for lenders with higher expected growth opportunities.

Lenders which have stronger growth prospects, i.e., a higher α , can use the future profits to reduce the spread on debt. A key feature of this prediction is that the lower spread is, in part, driven by higher effort provision. This is potentially testable. One can proxy lender monitoring and effort provision with borrower site visits, hiring third party appraisers, or the frequency with which lenders demand loan-specific information (see Gustafson et al. 2021).

The intuition is as follows: At t = 0, a higher α implies higher expected NPV on the second project. This has two effects: a direct one and an indirect one. The increase in the expected NPV (which is sold to the t = 0 investor) directly reduces the face value of debt, D_1 , in the first project due to a bigger reduction in this project's funding requirement. This fall in D_1 relaxes the effort moral hazard constraint in the first project and leads to higher effort provision which, in turn, reduces the bank's default probability, and hence, the spread on debt falls further.

2. The spread on debt falls in the bank's growth opportunities if these are positively correlated with its core activities. If they are negatively correlated, the effect may reverse.

If there is a positive correlation between the lender's growth opportunities, α , and the return on its assets-in-place, Y, then an increase in one is accompanied by an increase in the other. The effect of both changes are in the same direction, which is that the spread on debt falls. If on the other hand, Y and α are negatively correlated, then an increase in one is accompanied by a fall in the other. As a result, these two changes have opposite effects on the spread and which effect dominates depends on the parameters.

5 Conclusion

We present a model in which there are two investment opportunities for a bank; the first suffers from an effort moral hazard problem, while the second suffers from an adverse selection problem. We show that on-balance sheet funding with cross-subsidization across projects strictly dominates market funding, on-balance sheet funding without crosssubsidization and off-balance sheet financing with voluntary support for all parameter values, and delivers the second-best outcome. In our solution, expected profits from projects which suffer from adverse selection are used to induce higher effort provision in projects subject to effort moral hazard. Our analysis implies that, given the informational frictions in the present setting, the optimal solution requires on-balance sheet funding rather than off-balance sheet funding. Hence, in order to provide a rationale for voluntary support in off-balance sheet funding, we need to identify a different combination of frictions.

6 Appendix

6.1 Proofs

Proof of Lemma 1:

Proof. Due to limited liability, $D_{2,3} \leq R - D_{0,3}$, In a separating equilibrium in which bad banks obtain funding without pooling with good banks, investors' participation constraint is violated since for any $D_{2,3}$ satisfying the limited liability constraint (even for $D_{0,3} = 0$, and hence, for all feasible $D_{0,3}$), $p_b D_{2,3} < 0$ (see Assumption A1). Therefore, there cannot exist a separating equilibrium in which bad banks obtain funding.

Proof of Lemma 2:

Proof. Suppose that the equilibrium is pooling. In Figure 2a, the blue dashed line denotes the combination of K and $D_{2,3}$ such that investors break even in a pooling equilibrium. Investors provide funds in a pooling equilibrium for any combination of K and $D_{2,3}$ which lies above the blue dashed line. Note that the blue dashed line lies entirely above the zero profit line of bad banks, $\Pi_b = 0$ – bad banks participate on or below this line. This implies that for any K, the repayment that investors require to break even in a pooling equilibrium is such that it delivers negative profits for bad banks, which violates the rationality assumption. Thus, the conjectured equilibrium cannot exist.

Proof of Lemma 4:

Proof. Suppose that there exists a market breakdown equilibrium at t = 2. Given the beliefs associated with the breakdown equilibrium, the expected payoff that the t = 0 investors obtain from the second project is 0, which implies that the price that they are

willing to pay for this cash flow is Z = 0. In this case, the bank needs to raise the full 1 unit in order to invest in the moral hazard project. Thus, the face value of debt in the first project in this case is higher than in the case when it is expected that there will be funding for the good-type second project. From Lemma 3, for any K > 0 and $D_{0,3} \leq R - \frac{1}{p_q}$, the t = 2 investors can achieve separation in the shaded region in Figure 2a, such that they can offer funding in the good-type second project. Given this offer, good banks at t = 2 will invest since their participation constraint is satisfied, while bad banks will not invest since their participation constraint is violated in this region. By the Intuitive Criterion, investors assign a probability 1 to the event that only good banks will invest. Thus, there exists a profitable deviation from the K = 0 market breakdown equilibrium in which investors offer K > 0 which ensures that good projects obtain funding at t = 2. Anticipating that good banks will invest, the t = 0 investors are willing to purchase a portion of the second project for Z > 0, which reduces the bank's funding requirement for the first project and, in turn, reduces the promised repayment for the first project, D_1 . This lower D_1 will induce higher effort provision and lead to higher profits which the bank will (at least partly) capture. Therefore, this deviation is strictly profitable both for the bank and the investors, and hence, the market breakdown equilibrium cannot exist.

Proof of Lemma 5:

Proof. Suppose that K > 0 and $D_{0,3} < R - \frac{1}{p_g}$. It is clear from Equation (11) that bank profits are maximized when K takes its smallest value and $D_{0,3}$ takes its largest possible value. Hence, for any K > 0 and $D_{0,3} < R - \frac{1}{p_g}$, a new investor will enter the market to offer a smaller K and a higher $D_{0,3}$, and attract the bank profitably. The new contract will reduce the promised repayment for the first project, D_1 . A lower D_1 leads to higher effort in the first project and increases bank value, so the bank accepts the deviant offer. Thus, competition among investors sets $K^* = 0$ and $D_{0,3}^* = R - \frac{1}{p_g}$.

Proof of Proposition 4:

Proof. With regards to the adverse selection project, there is full separation between good and bad banks in both single bank and market funding cases. Thus, to show that single bank funding dominates market funding in terms of efficiency, we need to show that $e^* > e^D$ for all parameter values. First, we show that $D_1^* > D_1^D$. Using Equations 12 and 17:

$$D_1^* = \frac{1 - (1 - p_g - me)Y}{p_g + me} - \underbrace{\frac{\alpha(p_g R - 1)}{p_g + me}}_{>0} < \frac{1 - (1 - p_g - me)Y_1}{p_g + me} = D_1^D$$
(23)

Using $D_1^* < D_1^D$ and Equations 13 and 18:

$$c'(e^*) = m(R - D_1^* + Y) > m(R - D_1^D + Y) = c'(e^D)$$
(24)

Since c'(e) is increasing in e, it holds that $e^* > e^D$.

Proof of Lemma 6:

Proof. First we show that $D_1^* < D_1^{on} < D_1^{off}$. Noting that $1 - p_g - \alpha > me$ since it is assumed that $1 - p_g \ge me$ and using Equations 12, 19, and 21:

$$D_{1}^{*} = \frac{1 - (1 - p_{g} - me)Y}{p_{g} + me} - \underbrace{\frac{\alpha(p_{g}R - 1)}{p_{g} + me}}_{>0} < \underbrace{\frac{1 - (1 - p_{g} - me)Y}{p_{g} + me}}_{D_{1}^{on}}$$

$$= \underbrace{\frac{1 - (1 - p_{g} - \alpha - me)Y - \alpha Y}{p_{g} + me}}_{\text{add and subtract } \alpha Y} < \frac{1 - \alpha Y}{p_{g} + me} = D_{1}^{off}$$

$$(25)$$

Using $D_1^* < D_1^{on} < D_1^{off}$ and Equations 13, 20, and 22:

$$c'(e^*) = m(R - D_1^* + Y) > \underbrace{m(R - D_1^{on} + Y)}_{c'(e^{on})} > m(R - D_1^{off}) = c'(e^{off})$$
(26)

Since c'(e) is increasing in e, it holds that $e^* > e^{on} > e^{off}$.

6.2 Limited liability constraint at t = 2

In the main text, we consider the case the limited liability constraint of the bank at t = 2 is not binding. Below, we consider the case in which the limited liability constraint is binding (i.e., $D_{2,3} > R - D_{0,3}$). In this case, the participation constraint of an investor facing a type g bank becomes:

$$\gamma_g = p_g(R - D_{0,3}) - (1 - K) \ge 0 \tag{27}$$

When the participation constraint binds we get, $K \ge 1 - p_g(R - D_{0,3})$, which is the green vertical line in the $(K, D_{2,3})$ space in Figure 3. For an equilibrium to be feasible, it must lie on or to the right of this green vertical line. When the limited liability constraint is slack (see Figure 2a), this line is in the second quadrant and the green line intersects the horizontal axis at K < 0, and when it just binds (see Figure 2b), it is along the vertical axis. Thus, for $D_{0,3} \le R - \frac{1-K}{p_g}$, this line does not affect the analysis. When the limited liability constraint is binding, the vertical line appears in the first quadrant, ruling out some of the equilibria. The set of feasible funding equilibria is along the $\Pi_g = \gamma_g = 0$



Figure 3: Feasible equilibria, $D_{0,3} > R - \frac{1-K}{p_q}$

line, but on or to the right of the green line.

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